The matter's inertia and interaction in an isolated system Zhengfa Li*
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Abstract: The intrinsic inertia and interaction of matters with unchanged density in an isolated system are studied. It is shown that the matters have a proved inertia of spin with the angular velocity $\vec{\omega}_{\boldsymbol{n}}=\frac{d \theta}{d t} \widehat{\boldsymbol{n}}=\boldsymbol{\nabla} \times \overrightarrow{\boldsymbol{u}}$ of isolated system. The conclusive proof can reinforce Newton's the first law, taking into account the non-zero volume of point close to zero without limitation, and can explain the matter wave and seismic waves. A fundamental isolated system with two coupling matters has studied further. The revealed coupling characterizations have been used to explain the DNA structure, Time Cone and topological sphere of moving trace. The proven interaction within two matters is the coupling result of uniform rectilinear motion and spin of isolated system, which may be helpful to uniform the gravitation and electromagnetic force.

Introduction.-The conservation laws of matters can be classified as two distinct categories: those that are scalar including of mass, energy [1], electric charge; and those that are vector including of momentum, angular momentum, acceleration, angular acceleration, and spin of quantum particles as electron or atom [2,3]. Based on the viewpoint of the matters depending on ascertained space-time rather than mass point without any

[^0]volume [4], the systems could be analyzed as followed.
Model.-A infinitesimal even isolated system is used as the physical model, and its continuous equation of matters can be deduced using Lagrangian method. According to the conservation law, the matters of infinitesimal system keep constant as the product of density $\boldsymbol{\rho}$ and volume $\boldsymbol{V}, \boldsymbol{T}=\boldsymbol{\rho} \boldsymbol{V}=\boldsymbol{C}$, and its differential coefficient to the time parameter $\boldsymbol{t}$ is listed as follows:
\[

$$
\begin{equation*}
\left.\frac{\mathrm{d} \rho}{\mathrm{~d} t}\right|_{V}+\left.\rho \frac{\mathrm{d} V}{V \mathrm{~d} t}\right|_{\rho}=0 \tag{1}
\end{equation*}
$$

\]

The centre of infinitesimal even isolated system is set up as the origin of proprio-coordinate system to characterize its intrinsic properties, similarly to the centre of mass. The velocity of any point in the system is described by $\vec{u}=\frac{d \vec{l}}{d t}$, here $\mathrm{d} \vec{l}$ for the displacement vector within time quantum $\mathrm{d} t$, and its divergence noted firstly in Chinese by Mohist Canon is listed as follows,

$$
\begin{align*}
& \qquad \begin{aligned}
& \operatorname{div} \vec{u}=\frac{1}{V} \lim _{V \rightarrow 0} \oiint_{\partial V} \vec{u} \cdot d \vec{S}=\frac{1}{V} \lim _{V \rightarrow 0} \oiint_{\partial V}\left(\frac{d \vec{l}}{d t}\right) \cdot \mathrm{d} \vec{S} \\
&= \frac{1}{V d t} \lim _{V \rightarrow 0} \oiint_{\partial V} d \vec{l} \cdot d \vec{S} \\
&= \frac{1}{V d t} \lim _{V \rightarrow 0} \oiint_{\partial V} d(\vec{l} \cdot \vec{S})=\frac{1}{V d t} \lim _{V \rightarrow 0} V=\frac{d V}{V d t} \\
& \text { i. e. } \\
& \qquad \operatorname{div} \vec{u}=\frac{d V}{V d t}
\end{aligned} \quad \text { (2) }
\end{align*}
$$

Substitute equation (2) into equation (1), then

$$
\begin{equation*}
\left.\frac{d \rho}{d t}\right|_{V}+\left.\rho \operatorname{div} \vec{u}\right|_{\rho}=0 \tag{3}
\end{equation*}
$$

As the most important and fundamental theory during the transport
process in mechanics, the law of matter conservation with differential form is obtained, in which the density $\boldsymbol{\rho}$ or velocity $\boldsymbol{u}$ is variable for time $\boldsymbol{t}$ with constant volume $\boldsymbol{V}$ or constant density $\boldsymbol{\rho}$, respectively.

The following analysis can be conducted from equations (1-3):
Part ( I): Kinematics of a single homogeneous matter within an isolated system.- If there is a single homogeneous matter $T_{1}$ in the isolated system as shown in Dao De Jing, like deformable soft-body [5] water or rigid top, and its volume $V_{1}$ fills all space of the system [6].

$$
\begin{gathered}
T_{1}=\lim _{V_{1} \rightarrow 0} \rho_{1} V_{1}=\rho_{1} \lim _{V_{1} \rightarrow 0} V_{1} \\
\rho_{1}=\frac{T_{1}}{V_{1}}=\lim _{V_{1} \rightarrow 0} \frac{T_{1}}{V_{1}}=\frac{d T_{1}}{d V_{1}}=C_{1(t)}
\end{gathered}
$$

From equation (3), then

$$
\begin{equation*}
\left.\frac{d \rho_{1}}{d t}\right|_{V_{1}}=\left.\frac{d\left(\frac{d T_{1}}{d V_{1}}\right)}{d t}\right|_{V_{1}}=\left.\frac{d^{2} T_{1}}{d V_{1} d t}\right|_{V_{1}}=\left.\frac{d C_{(t)}}{d t}\right|_{V_{1}}=-\left.\rho_{1} \operatorname{div} \vec{u}_{1}\right|_{\rho_{1}} \tag{4}
\end{equation*}
$$

While $C_{1(t)}$ equals a non-zero and non-infinity constant to the physical meaning, $\left.\rho_{1} \operatorname{div} \vec{u}_{1}\right|_{\rho_{1}}=0$ or div $\left.\vec{u}_{1}\right|_{\rho_{1}}=0$ can be obtained from equation (4), and integrated further as

$$
\begin{equation*}
\vec{u}_{1}=\vec{C}^{\prime}\left(C^{\prime} \neq \pm \infty\right) \tag{5}
\end{equation*}
$$

While the single homogeneous matter $T$ of the system isn't fill space $V$ fully, the rest space has the volume $V^{\prime}$, density $\rho^{\prime}=0$, matter $T^{\prime}=0$ and velocity $\vec{u}^{\prime}=0$. So, $T=T_{1}+T^{\prime}=T_{1} \neq 0, V=V_{1}+V^{\prime} \neq 0$ and from equation (3),

$$
\rho_{1}=\frac{T_{1}}{V_{1}}=\lim _{V_{1} \rightarrow 0} \frac{T_{1}}{V_{1}}=\frac{d T_{1}}{d V_{1}}=C_{1(t)}
$$

$$
\begin{gathered}
\rho^{\prime}=\frac{T^{\prime}}{V^{\prime}}=\lim _{V^{\prime} \rightarrow 0} \frac{T^{\prime}}{V^{\prime}}=\frac{d T^{\prime}}{d V^{\prime}}=C_{(\mathrm{t})}^{\prime}=0 \\
\bar{\rho}=\frac{T}{V}=\frac{T_{1}+T^{\prime}}{V}=\frac{\rho_{1} V_{1}+\rho^{\prime} V^{\prime}}{V}=\frac{C_{1(t)} V_{1}+C_{(t)}^{\prime} V^{\prime}}{V}=\frac{V_{1}}{V} C_{1(t)}=\frac{V_{1}}{V} \rho_{1}
\end{gathered}
$$

For the system,

$$
\begin{gather*}
\left.\frac{d \bar{\rho}}{d t}\right|_{V}+\left.\bar{\rho} \operatorname{div} \overline{\vec{u}}\right|_{\overline{\bar{\rho}}}=0 \\
\left.\frac{d T}{d t}\right|_{V}+\left.T \operatorname{div} \overline{\vec{u}}\right|_{\bar{\rho}}=0 \\
\left.\because \frac{d T}{d t}\right|_{V}=\left.0 \quad \therefore \operatorname{div} \overline{\vec{u}}\right|_{\bar{\rho}}=-\left.\frac{d T}{T d t}\right|_{V}=0 \\
\text { Therefore } \quad \overline{\vec{u}}=\frac{\vec{u}_{1} T_{1}+\vec{u}^{\prime} T^{\prime}}{T_{1}+T^{\prime}}=\vec{C}^{\prime}=\vec{u}_{1}\left(C^{\prime} \neq \pm \infty\right) \tag{5}
\end{gather*}
$$

Above equation revealed that the velocity of even matter keeps the same constant as the average velocity of the isolated system with single homogenous matter filled fully or not. If the matter is mass, the total momentum for the system keeps invariantly with the product of its mass and velocity.

Suppose that $\overrightarrow{\boldsymbol{l}}_{\mathbf{1}}=l_{1} \hat{\boldsymbol{r}}$ at the moment $\boldsymbol{t}$, and in polar coordinate system with the same centre,

$$
\begin{align*}
\stackrel{\rightharpoonup}{\boldsymbol{u}}_{1} & =\frac{d \overrightarrow{\boldsymbol{l}}_{\mathbf{1}}}{d t}=\frac{d l_{1}}{d t} \hat{\boldsymbol{r}}+l_{1} \frac{d \hat{\boldsymbol{r}}}{d t}=\frac{d l_{1}}{d t} \hat{\boldsymbol{r}}+l_{1} \frac{d \boldsymbol{\theta}}{d t} \hat{\boldsymbol{\tau}}=\frac{d l_{1}}{d t} \hat{\boldsymbol{r}}+\omega_{n} l_{1} \hat{\boldsymbol{v}} \\
& =\overrightarrow{\boldsymbol{u}}_{r}+\overrightarrow{\boldsymbol{u}}_{\tau}+\overrightarrow{\boldsymbol{u}}_{n}=\overrightarrow{\boldsymbol{u}}_{r}+\overrightarrow{\boldsymbol{u}}_{\tau} \quad\left(\overrightarrow{\boldsymbol{u}}_{n}=0 \widehat{\boldsymbol{n}}=0\right) \tag{6}
\end{align*}
$$

Here the displacement vector $\hat{\boldsymbol{r}}$ is the unit vector for any point to the centre of system; vector $\hat{\boldsymbol{\tau}}$ is the unit vector perpendicular to the $\hat{\boldsymbol{r}}$ and lies in the plane consisted by the centre of system and unit vector $\hat{\boldsymbol{r}}, \hat{\boldsymbol{\tau}} \perp \hat{\boldsymbol{r}}$;
$\overrightarrow{\boldsymbol{\omega}}_{n}=\frac{d \theta}{d t} \widehat{\boldsymbol{n}}=\boldsymbol{\nabla} \times \overrightarrow{\boldsymbol{u}}_{1}$ is the angular velocity of spin and its direction lines in unit vector $\widehat{\boldsymbol{n}}$ which is perpendicular to unit vectors $\hat{\boldsymbol{\tau}}$ and $\hat{\boldsymbol{r}}$, simultaneously.

$$
\left.\begin{aligned}
& \left|\overrightarrow{\boldsymbol{u}}_{1}\right|=\sqrt{\left(\frac{d l_{1}}{d t}\right)^{2}+\left(l_{1} \frac{d \boldsymbol{\theta}}{d t}\right)^{2}}=\left|\vec{C}^{\prime}\right| \\
& \quad=\left\lvert\, \sqrt{\left(\frac{d l_{1}}{d t}\right)^{2}+\left(l_{1} \frac{d \boldsymbol{\theta}}{d t}\right)^{2}}\left[\frac{\frac{d l_{1}}{d t}}{\sqrt{\left(\frac{d l_{1}}{d t}\right)^{2}+\left(l_{1} \frac{d \boldsymbol{\theta}}{d t}\right)^{2}}} \hat{\boldsymbol{r}}\right.\right. \\
& \\
& \quad+\frac{l_{1} \frac{d \boldsymbol{\theta}}{d t}}{\sqrt{\left(\frac{d l_{1}}{d t}\right)^{2}+\left(l_{1} \frac{d \boldsymbol{\theta}}{d t}\right)^{2}}} \hat{\boldsymbol{\tau}}
\end{aligned} \right\rvert\,
$$

Given $\alpha$ as the angle between $\overrightarrow{\boldsymbol{u}}_{1}$ and $\hat{\boldsymbol{r}}$ is equal to the product of $\omega_{n}$ and some parameter $\Delta$, then

$$
\begin{aligned}
& \sin \alpha=\frac{l_{1} \frac{d \theta}{d t}}{\sqrt{\left(\frac{d l_{1}}{d t}\right)^{2}+\left(l_{1} \frac{d \theta}{d t}\right)^{2}}}, \cos \alpha=\frac{\frac{d l_{1}}{d t}}{\sqrt{\left(\frac{d l_{1}}{d t}\right)^{2}+\left(l_{1} \frac{d \theta}{d t}\right)^{2}}} \\
& \tan \alpha=\frac{l_{1} \frac{d \theta}{d t}}{\frac{d l_{1}}{d t}}=l_{1} \frac{d \theta}{d l_{1}}, \alpha=\tan ^{-1}\left(l_{1} \frac{d \theta}{d l_{1}}\right)=\omega_{n} \Delta
\end{aligned}
$$

$$
\text { Thus } \begin{align*}
& \overrightarrow{\boldsymbol{u}}_{1}=\sqrt{\left(\frac{d l_{1}}{d t}\right)^{2}+\left(l_{1} \frac{d \boldsymbol{\theta}}{d t}\right)^{2}}[(\cos \alpha) \hat{\boldsymbol{r}}+(\sin \alpha) \hat{\boldsymbol{\tau}}] \\
&=A \omega_{n}\left[\left(\cos \omega_{n} \Delta\right) \hat{\boldsymbol{r}}+\left(\sin \omega_{n} \Delta\right) \hat{\boldsymbol{\tau}}\right]=\vec{C}^{\prime} \tag{7}
\end{align*}
$$

Here $A=\sqrt{\left(\frac{d l_{1}}{d \theta}\right)^{2}+l_{1}^{2}}$, the velocity $\overrightarrow{\boldsymbol{u}}_{1}$ can be identified as the wave with spin of $\omega_{n}$ in the space field.

Discussions:

1 )If $l_{1}$ keeps constant, then $\alpha=90^{\circ}, \overrightarrow{\boldsymbol{u}}_{1} \perp \hat{\boldsymbol{r}}, \overrightarrow{\boldsymbol{u}}_{1} / / \hat{\boldsymbol{\tau}}$, the circling motion of matter can be observed on an axis lining with unit vector $\widehat{\boldsymbol{n}}$ and crossing the centre of system. Considering the space volume of system is close to zero without limitation, $\mathrm{V} \rightarrow 0$ and $\mathrm{V} \neq 0$, the conclusion of the spin of matter can drown with uniform angular velocity.

$$
\begin{equation*}
\overrightarrow{\boldsymbol{u}}_{1}=\overrightarrow{\boldsymbol{u}}_{\tau}=l_{1} \frac{d \boldsymbol{\theta}}{d t} \hat{\boldsymbol{\tau}}=\omega_{n} l_{1} \hat{\boldsymbol{\tau}}=\vec{C}^{\prime} \tag{8}
\end{equation*}
$$

2 ) If $\boldsymbol{\theta}$ keeps constant, then $\alpha=0^{0}, \overrightarrow{\boldsymbol{u}}_{1} \perp \hat{\boldsymbol{\tau}}, \overrightarrow{\boldsymbol{u}}_{1} / / \hat{\boldsymbol{r}}$, the uniform rectilinear motion of matter can be observed lining with unit vector $\hat{\boldsymbol{r}}$ and crossing the centre of system.

$$
\begin{equation*}
\overrightarrow{\boldsymbol{u}}_{1}=\overrightarrow{\boldsymbol{u}}_{r}=\frac{d \overrightarrow{\boldsymbol{l}}_{1}}{d t}=\frac{d l_{1}}{d t} \hat{\boldsymbol{r}}=\vec{C}^{\prime} \tag{9}
\end{equation*}
$$

3 ) If $l_{1}$ and $\boldsymbol{\theta}$ keep constant, then the matter of system keep static with

$$
\begin{equation*}
\overrightarrow{\boldsymbol{u}}_{1}=0 . \tag{10}
\end{equation*}
$$

Supposing the matter is mass, the contents of equations (8), (9) and (10) are the proved and reinforced Newton's the first law [4], not for mass point with zero volume in space, but for the single homogeneous system with its non-zero volume close to zero with no limitation, and its density keeps a constant of neither zero nor infinity. The inertia of motive matter of an isolated homogeneous system including quantum particle, is to keep static state, uniform rectilinear motion or uniform angular spin [7].

4 )If $l_{1}$ and $\boldsymbol{\theta}$ are all changing, and $\alpha$ may be some ascertained value of $0^{\circ}$ to $90^{\circ}$, the matter performs the collective motive state of uniform rectilinear motion and uniform angular spin along the directions of $\hat{\boldsymbol{r}}$
and $\overrightarrow{\boldsymbol{\tau}}$, respectively, liking two types of seismic waves. As expressed in the equation of (7), the velocity $\overrightarrow{\boldsymbol{u}}_{1}$ shows the motion of polarized transverse wave, where the properties of wave can be explained as the extension of uniform angular spin along the direction of uniform rectilinear motion, and its properties of particle can be explained as the uniform rectilinear motion, as the same as the matter wave in de Broglie hypothesis [8].

Part (II): Kinematics and dynamics of two homogeneous matters
within an isolated system.-If two homogeneous matters $T_{1}$ and $T_{2}$ in an isolated system as noted in The Book of Changes, their volumes $V_{1}$ and $V_{2}$ are not filled all space of the system. At the initial time $t_{0}$, an the origin $\boldsymbol{O}$ of proprio-coordinate system is signed by the centre of infinitesimal isolated system, and it will change to $\boldsymbol{O}_{1}$ at time $t_{1}$ and further change to $\boldsymbol{O}_{d}$ at time $t$ with a range of $d t=t-t_{1}$. As for the spins of matters $T_{1}$ and $T_{2}$ can be ignored, considering that the supposed distances between matters and the centre of isolated system are very larger than their dimensions, all the movement about the integral system is discussed for the two-point model of matters $T_{1}$ and $T_{2}$ as shown in Dao De Jing. For the matter point of $T_{1}$, its location can be described as a vector $\vec{r}_{11}$ to the centre of $\boldsymbol{O}_{1}$ and anther vector $\vec{r}_{1}$ to the centre of $\boldsymbol{O}_{d}$. The similar vectors $\vec{r}_{21}$ and $\vec{r}_{2}$ can be listed for matter point of $T_{2}$. Therefore, the points of centres $\boldsymbol{O}, \boldsymbol{O}_{1}$ and $\boldsymbol{O}_{\boldsymbol{d}}$ can mark a plane of $\overline{\boldsymbol{O O}_{\mathbf{1}} \boldsymbol{O}_{\boldsymbol{d}}}, \vec{l}=\overrightarrow{O_{1}}$ and $d \vec{l}=\overline{O_{1} O_{\mathrm{d}}}$ are defined
as the displacement and unit one of the whole isolated system, as shown in Fig. 1. The characters of matters $T_{1}, T_{2}$ and rest space in the isolated system are shown as $\left(T_{1}, V_{1}, \rho_{1}, \vec{r}_{11}, \vec{r}_{1}, \vec{u}_{1} ; T_{2}, V_{2}, \rho_{2}, \vec{r}_{21}, \vec{r}_{2}, \vec{u}_{2} ;\right.$ $T_{3}=0, V_{3}, \rho_{3}=0, \vec{u}_{3}=0$ ), and their relationships are $T=T_{1}+T_{2}+T_{3}=T_{1}+T_{2}=C \neq 0$ and $\boldsymbol{V}=\boldsymbol{V}_{\mathbf{1}}+\boldsymbol{V}_{\mathbf{2}}+\boldsymbol{V}_{\mathbf{3}} \neq 0$.

## FIG. 1. The geometric graph of moving matters $T_{1}$ and $T_{2}$ in an isolated system.

 Therefore, from equation (3),$$
\begin{gathered}
\rho_{1}=\frac{T_{1}}{V_{1}}=\lim _{V \rightarrow 0} \frac{T_{1}}{V_{1}}=\frac{d T_{1}}{d V_{1}}=C_{1(t)} \\
\rho_{2}=\frac{T_{2}}{V_{2}}=\lim _{V \rightarrow 0} \frac{T_{2}}{V_{2}}=\frac{d T_{2}}{d V_{2}}=C_{2(t)} \\
\rho_{3}=\frac{T_{3}}{V_{3}}=\lim _{V \rightarrow 0} \frac{T_{3}}{V_{3}}=\frac{d T_{3}}{d V_{3}}=C_{3(t)}=0 \\
\bar{\rho}=\frac{T}{V}=\frac{T_{1}+T_{2}+T_{3}}{V}=\frac{\rho_{1} V_{1}+\rho_{2} V_{2}+\rho_{3} V_{3}}{V}=\frac{C_{1(t)} V_{1}+C_{2(t)} V_{2}}{V}
\end{gathered}
$$

Put above parameters into the equation (3) for all isolated system

$$
\begin{gather*}
\left.\frac{d \bar{\rho}}{d t}\right|_{V}+\left.\bar{\rho} \operatorname{div} \overline{\vec{u}}\right|_{\rho}=0 \\
\left.\frac{d\left(T_{1}+T_{2}\right)}{d t}\right|_{V}+\left.\left(T_{1}+T_{2}\right) \operatorname{div} \overline{\vec{u}}\right|_{\rho}=0 \\
\left.\left(T_{1}+T_{2}\right) \operatorname{div} \overline{\vec{u}}\right|_{\rho}=0 \\
\overrightarrow{\vec{u}}=\vec{C}^{\prime}\left(C^{\prime} \neq \pm \infty\right) \tag{5}
\end{gather*}
$$

Above equation implied that the average velocity $\overline{\vec{u}}$ of two-matter isolated system keeps constant vector $\vec{C}^{\prime}$. If the matters are mass, the total momentum $\vec{P}$ of the system also keeps invariantly named the
conservation law of momentum as listed.

$$
\begin{equation*}
\vec{P}=T_{1} \vec{u}_{1}+T_{2} \vec{u}_{2}=\left(T_{1}+T_{2}\right) \overline{\vec{u}}=\left(T_{1}+T_{2}\right) \vec{C}^{\prime} \quad\left(C^{\prime} \neq \pm \infty\right) \tag{11}
\end{equation*}
$$

And the average velocity of the whole isolated system is

$$
\begin{gathered}
\overline{\overline{\boldsymbol{u}}}=\frac{T_{1} \vec{u}_{1}+T_{2} \vec{u}_{2}}{T_{1}+T_{2}}=\frac{T_{1} \vec{u}_{1}}{T_{1}+T_{2}}+\frac{T_{2} \vec{u}_{2}}{T_{1}+T_{2}}=\vec{C}^{\prime} \\
T_{1} \vec{u}_{1}+T_{2} \vec{u}_{2}=\left(T_{1}+T_{2}\right) \vec{C}^{\prime}\left(C^{\prime} \neq \pm \infty\right)
\end{gathered}
$$

As the same as equation (6), here $\overline{\overline{\boldsymbol{u}}}$ presents the collective motive state of uniform rectilinear motion and angular spin of the whole system's centre. Get derivative with respect to time, above equation comes into

$$
\frac{d\left(T_{1} \vec{u}_{1}+T_{2} \vec{u}_{2}\right)}{d t}=T_{1} \frac{d \vec{u}_{1}}{d t}+T_{2} \frac{d \vec{u}_{2}}{d t}=\frac{d\left(T_{1}+T_{2}\right) \vec{C}^{\prime}}{d t}=\left(T_{1}+T_{2}\right) \frac{d \vec{C}^{\prime}}{d t}
$$

Define $\vec{a}_{1}=\frac{d \vec{u}_{1}}{d t}$ and $\vec{a}_{2}=\frac{d \vec{u}_{2}}{d t}$, then

$$
T_{1} \vec{a}_{1}+T_{2} \vec{a}_{2}=0
$$

Further define $\vec{F}_{1}=T_{1} \vec{a}_{1}$ and $\vec{F}_{2}=T_{2} \vec{a}_{2}$, similarly as Newton's the second Law [4], therefore $\vec{F}_{1}+\vec{F}_{2}=0$ i.e. $\vec{F}_{1}=-\vec{F}_{2}$ has obtained as Newton's the third Law [4].

Based on the system centre $\boldsymbol{O}_{\mathbf{1}}$, the respective proper orthogonal decompositions of $\vec{u}_{1}, \vec{u}_{2}, \overline{\overline{\boldsymbol{u}}}$, and $\vec{C}^{\prime}$ vectors along $\hat{\boldsymbol{r}}, \hat{\boldsymbol{\tau}}$, and $\widehat{\boldsymbol{n}}$ unit vectors, can be substituted into Equation (11) as followed.

$$
\begin{align*}
\vec{P} & =T_{1}\left(\vec{u}_{1 r}+\vec{u}_{1 \tau}+\vec{u}_{1 n}\right)+T_{2}\left(\vec{u}_{2 r}+\vec{u}_{2 \tau}+\vec{u}_{2 n}\right) \\
& =\left(T_{1}+T_{2}\right)\left(\overline{\vec{u}}_{r}+\overline{\vec{u}}_{\tau}+\overrightarrow{\vec{u}}_{n}\right) \\
& =\left(T_{1}+T_{2}\right)\left(\vec{C}_{r}^{\prime}+\vec{C}_{\tau}^{\prime}+\vec{C}_{n}^{\prime}\right) \quad\left(C^{\prime} \neq \pm \infty\right) \tag{11'}
\end{align*}
$$

Therefore, the components of system along the $\hat{\boldsymbol{r}}, \hat{\boldsymbol{\tau}}$ and $\widehat{\boldsymbol{n}}$ directions
are shown, following the same conservation law

$$
\begin{align*}
& \vec{P}_{r}=T_{1} \vec{u}_{1 r}+T_{2} \vec{u}_{2 r}=\left(T_{1}+T_{2}\right) \overline{\vec{u}}_{r}=\left(T_{1}+T_{2}\right) \vec{C}_{r}^{\prime}  \tag{11-1}\\
& \vec{P}_{\tau}=T_{1} \vec{u}_{1 \tau}+T_{2} \vec{u}_{2 \tau}=\left(T_{1}+T_{2}\right) \overline{\vec{u}}_{\tau}=\left(T_{1}+T_{2}\right) \vec{C}_{\tau}^{\prime}  \tag{11-2}\\
& \vec{P}_{n}=T_{1} \vec{u}_{1 n}+T_{2} \vec{u}_{2 n}=\left(T_{1}+T_{2}\right) \stackrel{\rightharpoonup}{u}_{n}=\left(T_{1}+T_{2}\right) \vec{C}_{n}^{\prime} \tag{11-3}
\end{align*}
$$

For $\vec{u}_{n}$ is zero in equation (6), the equation (11-3) is transformed to

$$
\begin{equation*}
\vec{P}_{n}=T_{1} \vec{u}_{1 n}+T_{2} \vec{u}_{2 n}=0 \tag{11-3'}
\end{equation*}
$$

and further analyzed as, that the centre of two-matter isolated system has no translation or rotation, but only harmonic vibration of two matters as two spring oscillators around the centre and along direction $\widehat{\boldsymbol{n}}$ for astringency, their velocities follow the equation as

$$
\begin{equation*}
\vec{u}_{1 n}=-\frac{T_{2}}{T_{1}} \vec{u}_{2 n} \tag{12-1}
\end{equation*}
$$

and it performs the collective motive state of uniform rectilinear motion and uniform angular spin in the plane of $\overline{\boldsymbol{O O}_{\mathbf{1}} \boldsymbol{O}_{\boldsymbol{d}}}$ consisted by $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\tau}}$ unit vectors. Therefore, the translation, spin and vibration are orthogonal along the $\hat{\boldsymbol{r}}, \hat{\boldsymbol{\tau}}$ and $\widehat{\boldsymbol{n}}$ directions, respectively.

And the similar equations to equation (12-1) can be deduced as

$$
\begin{align*}
& \vec{u}_{1 r}=-\frac{T_{2}}{T_{1}} \vec{u}_{2 r}+\left(1+\frac{T_{2}}{T_{1}} \vec{C}_{r}^{\prime}\right)  \tag{12-2}\\
& \vec{u}_{1 \tau}=-\frac{T_{2}}{T_{1}} \vec{u}_{2 \tau}+\left(1+\frac{T_{2}}{T_{1}} \vec{C}_{\tau}^{\prime}\right) \tag{12-3}
\end{align*}
$$

If the angles between directions of vector $\hat{r}, \hat{\tau}$ or $\hat{n}$, and the line crossing two matters and the centre of isolated system, are defined as $\alpha, \beta$ or $\gamma \quad(\alpha, \beta$ or $\gamma \in[0, \pi / 2])$, the projection in direction $\hat{r}$ of vector $\vec{r}_{1}$ is $\vec{r}_{1 \perp}=\vec{r}_{1} \cos \alpha$. Considering the same angular velocity $\vec{\omega}_{n}$ of spin and
the same line of $\vec{r}_{1}$ and $\vec{r}_{2}$ with the opposite direction for the centre of two-mater isolated system, the directions of velocities $\vec{u}_{1 \tau}$ and $\vec{u}_{2 \tau}$ are opposite, and their values fit the equation $\vec{u}_{\tau}=r_{\perp} \omega_{n} \hat{\tau}$ from $\vec{u}_{\tau}=\vec{\omega}_{n} \times$ $\vec{r}_{\perp}$, and the equation (11-2) is transformed to

$$
\begin{align*}
& \qquad \begin{aligned}
& \vec{P}_{\tau}=T_{1} r_{1 \perp} \omega_{n} \hat{\tau}-T_{2} r_{2 \perp} \omega_{n} \hat{\imath}=\left(T_{1} r_{1 \perp}-T_{2} r_{2 \perp}\right) \omega_{n} \hat{\tau} \\
&=\left(T_{1} r_{1} \cos \alpha-T_{2} r_{2} \cos \alpha\right) \omega_{n} \hat{\tau}=\left(T_{1} r_{1}-T_{2} r_{2}\right) \omega_{n} \cos \alpha \hat{\tau} \\
&=\left(T_{1}+T_{2}\right) \vec{C}_{\tau}^{\prime}=\left(T_{1}+T_{2}\right) C_{\tau}^{\prime} \hat{\tau}
\end{aligned} \\
& \text { Therefore } \quad \omega_{n}=\frac{\left(T_{1}+T_{2}\right) c_{\tau}^{\prime}}{\left(T_{1} r_{1}-T_{2} r_{2}\right) \cos \alpha}
\end{align*}
$$

## Discussions:

1) When $\alpha=\pi / 2, \vec{P}_{\tau}=0, \vec{C}_{\tau}^{\prime}=0$, the equation (11-2) changes into the same form as equation (11-3), and equation (12-4) is to be meaningless. The collective motive states of polarizing motion with contrary phases both in $\hat{n}$ and $\hat{\tau}$ directions, and of motionless or uniform rectilinear motion in $\hat{r}$ direction, should shape the moving traces similar to the figures of Lissajous or twisted vines of plants [9].
$2)$ When $\alpha \neq \pi / 2, \omega_{n}=\frac{\left(\Delta T_{1}+\Delta T_{2}\right) C_{t}^{\prime}}{\left(\Delta T_{1} r_{1}-\Delta T_{2} r_{2}\right) \cos \alpha}$, especially for $\alpha=0$, then $\cos \alpha=1$ and the minimum value $\omega_{n}=\frac{\left(\Delta T_{1}+\Delta T_{2}\right) C_{\tau}^{\prime}}{\left(\Delta T_{1} r_{1}-\Delta T_{2} r_{2}\right)}$. The collective motive states of polarizing motion with contrary phases in direction $\hat{n}$, of spin with uniform the same angular velocity $\vec{\omega}_{n}$ in direction $\hat{\tau}$, and of motionless or uniform rectilinear motion in direction $\hat{r}$, should shape the moving traces similar to the familiar figures of double helix like DNA structure which physical mechanism has not been revealed
before [10] or of funnel shape like Time Cone revealed by Einstein [11].

3 ) When $\vec{C}_{r}^{\prime}=\vec{C}_{\tau}^{\prime}=0$, three equations of $(11-1,2,3)$ are in the same formation returning to equation (11) after vector overlay operation. The collective motive states of polarizing motion with contrary phases in $\hat{n}, \hat{\tau}$ and $\hat{r}$ directions should shape the moving trace of topological sphere, and the spins with contrary direction of two matters are both perpendicular to the surface of sphere [12].

4 ) Get derivative with respect to time, equations (12-1,2,3) become

$$
\begin{equation*}
\vec{a}_{1 r}=-\frac{T_{2}}{T_{1}} \stackrel{\rightharpoonup}{a}_{2 r} ; \quad \vec{a}_{1 \tau}=-\frac{T_{2}}{T_{1}} \stackrel{\rightharpoonup}{a}_{2 \tau} ; \vec{a}_{1 n}=-\frac{T_{2}}{T_{1}} \vec{a}_{2 n} \tag{13}
\end{equation*}
$$

The respective proper orthogonal decomposed equations (13) announce the relationship of $\vec{a}_{1}=-\frac{T_{2}}{T_{1}} \vec{a}_{2}$ in the line of matters $T_{1}$ and $T_{2}$, based on the centre $\boldsymbol{O}_{\mathbf{1}}$ of isolated system rather than that of matter $T_{1}$ or $T_{2}$.

From equation (6), the velocities of matters $T_{1}$ and $T_{2}$ are listed as

$$
\begin{align*}
\stackrel{\rightharpoonup}{\boldsymbol{u}}_{1} & =\frac{d \overrightarrow{\boldsymbol{r}}_{1}}{d t}=\frac{d r_{1}}{d t} \hat{\boldsymbol{r}}_{1}+r_{1} \frac{d \hat{\boldsymbol{r}}_{1}}{d t}=\frac{d r_{1}}{d t} \hat{\boldsymbol{r}}_{1}+r_{1} \frac{d \boldsymbol{\theta}}{d t} \hat{\boldsymbol{r}}_{1}=\frac{d r_{1}}{d t} \hat{\boldsymbol{r}}_{1}+\omega_{n} r_{1} \hat{\boldsymbol{\tau}}_{1} \\
& =\overrightarrow{\boldsymbol{u}}_{r 1}+\overrightarrow{\boldsymbol{u}}_{\tau 1}+\overrightarrow{\boldsymbol{u}}_{n 1}=\overrightarrow{\boldsymbol{u}}_{r 1}+\overrightarrow{\boldsymbol{u}}_{\tau 1} \quad\left(\overrightarrow{\boldsymbol{u}}_{n 1}=0 \widehat{\boldsymbol{n}}_{1}=0\right) \quad(6-1)  \tag{6-1}\\
\overrightarrow{\boldsymbol{u}}_{2} & =\frac{d \overrightarrow{\boldsymbol{r}}_{2}}{d t}=\frac{d r_{2}}{d t} \hat{\boldsymbol{r}}_{2}+r_{2} \frac{d \hat{\boldsymbol{r}}_{2}}{d t}=\frac{d r_{2}}{d t} \hat{\boldsymbol{r}}_{2}+r_{2} \frac{d \boldsymbol{\theta}}{d t} \hat{\boldsymbol{r}}_{2}=\frac{d r_{2}}{d t} \hat{\boldsymbol{r}}_{2}+\omega_{n} r_{2} \hat{\boldsymbol{\tau}}_{2} \\
= & \overrightarrow{\boldsymbol{u}}_{r 2}+\overrightarrow{\boldsymbol{u}}_{\tau 2}+\overrightarrow{\boldsymbol{u}}_{n 2}=\overrightarrow{\boldsymbol{u}}_{r 2}+\overrightarrow{\boldsymbol{u}}_{\tau 2} \quad\left(\overrightarrow{\boldsymbol{u}}_{n 2}=0 \widehat{\boldsymbol{n}}_{2}=0\right) \tag{6-2}
\end{align*}
$$

Their accelerations are listed as

$$
\overrightarrow{\boldsymbol{a}}_{\mathbf{1}}=\frac{d \overrightarrow{\boldsymbol{u}}_{1}}{d t}=\left[\frac{d^{2} r_{1}}{d t^{2}}-r_{1} \omega_{n 1}^{2}\right] \hat{\boldsymbol{r}}_{1}+\left(2 \omega_{n 1} \frac{d r_{1}}{d t}+r_{1} \frac{d \omega_{n 1}}{d t}\right) \hat{\boldsymbol{\tau}}_{1}+0 \widehat{\boldsymbol{n}}_{1}
$$

1

$$
\overrightarrow{\boldsymbol{a}}_{\mathbf{2}}=\frac{d \overrightarrow{\boldsymbol{u}}_{2}}{d t}=\left[\frac{d^{2} r_{2}}{d t^{2}}-r_{2} \omega_{n 2}^{2}\right] \hat{\boldsymbol{r}}_{2}+\left(2 \omega_{n 2} \frac{d r_{2}}{d t}+r_{2} \frac{d \omega_{n 2}}{d t}\right) \hat{\boldsymbol{\tau}}_{2}+0 \widehat{\boldsymbol{n}}_{2}
$$

The relationship of unit vectors ( $\hat{\boldsymbol{r}}_{1}, \hat{\boldsymbol{\tau}}_{1}, \widehat{\boldsymbol{n}}_{1}$ ) and ( $\hat{\boldsymbol{r}}_{2}, \hat{\boldsymbol{\tau}}_{2}, \widehat{\boldsymbol{n}}_{2}$ ) has been listed as followed,

$$
\begin{gathered}
\hat{\boldsymbol{r}}_{2}=-\hat{\boldsymbol{r}}_{1} \\
\hat{\boldsymbol{\tau}}_{2}=-\hat{\boldsymbol{\tau}}_{1} \\
\widehat{\boldsymbol{n}}_{2}=\widehat{\boldsymbol{n}}_{1}
\end{gathered}
$$

Considering the isolated system has the same angular velocity with that of matters $T_{1}$ and $T_{2}$, therefore, those angular velocities have the same value $\omega_{n 1}=\omega_{n 2}=\omega_{n}$ of binary system [13]. The forces $\overrightarrow{\boldsymbol{F}}_{1}$ and $\overrightarrow{\boldsymbol{F}}_{2}$ along and/or across the line between matters $T_{1}$ and $T_{2}$ are

$$
\begin{aligned}
& \overrightarrow{\boldsymbol{F}}_{\mathbf{1}}=\boldsymbol{T}_{\mathbf{1}} \overrightarrow{\boldsymbol{a}}_{\mathbf{1}}=\boldsymbol{T}_{\mathbf{1}}\left[\frac{d^{2} r_{1}}{d t^{2}}-r_{1} \omega_{n 1}^{2}\right] \hat{\boldsymbol{r}}_{1}+\boldsymbol{T}_{\mathbf{1}}\left(2 \omega_{n 1} \frac{d r_{1}}{d t}+r_{1} \frac{d \omega_{n 1}}{d t}\right) \hat{\boldsymbol{\tau}}_{1} \\
& \stackrel{\boldsymbol{F}}{2}=\boldsymbol{T}_{2} \stackrel{\boldsymbol{a}}{2}=\boldsymbol{T}_{2}\left[\frac{d^{2} r_{2}}{d t^{2}}-r_{2} \omega_{n 2}^{2}\right] \hat{\boldsymbol{r}}_{2}+\boldsymbol{T}_{2}\left(2 \omega_{n 2} \frac{d r_{2}}{d t}+r_{2} \frac{d \omega_{n 2}}{d t}\right) \hat{\boldsymbol{\tau}}_{2}
\end{aligned}
$$

With the formula $|\boldsymbol{\nabla} \times \overrightarrow{\boldsymbol{u}}|$ to instead of $\omega_{n}$, the uniformity from the centre of isolated system is

$$
\begin{align*}
\stackrel{\rightharpoonup}{\boldsymbol{F}}=\boldsymbol{T} \stackrel{\rightharpoonup}{\boldsymbol{a}} & =\boldsymbol{T}\left[\frac{d^{2} r}{d t^{2}}-r \omega_{n}^{2}\right] \hat{\boldsymbol{r}}+\boldsymbol{T}\left(2 \omega_{n} \frac{d r}{d t}+r \frac{d \omega_{n}}{d t}\right) \hat{\boldsymbol{\tau}} \\
& =\boldsymbol{T}\left[\frac{d^{2} r}{d t^{2}}-r|\nabla \times \vec{u}|^{2}\right] \hat{\boldsymbol{r}}+\boldsymbol{T}\left(2|\nabla \times \vec{u}| \frac{d r}{d t}+r \frac{d|\nabla \times \vec{u}|}{d t}\right) \hat{\boldsymbol{\tau}} \tag{14}
\end{align*}
$$

The previous equation (14) reveals that the interaction between two matters in composed isolated system has existed along and across the radius at the same time, especially containing attractive force [14]. And the equation (14) can be changed into quantum form, if the vectors of $\vec{r}$
and $\vec{u}$ have been changed into wave functions. As the simplest fundamental model of the motive matters' interaction in view of spin identified, it may be helpful to unite the four interactions in the field of space and velocity, which needs to be proved by experiments in future.

Conclusions.- The inertia and interaction have been studied for the matters with unchanged density in an isolated system. The inertia of spin with the angular velocity $\vec{\omega}_{n}=\frac{d \theta}{d t} \widehat{\boldsymbol{n}}=\boldsymbol{\nabla} \times \overrightarrow{\boldsymbol{u}}$ has been revealed and added into the Newton's the first law, for the matter of isolated system, by considering the point with non-zero volume but close to zero. The inertias of uniform angular spin and uniform rectilinear motion can be used to explain wave-particle duality and seismic waves. Further, the coupling isolated system with double matters has been chosen as the second fundamental model. The coupling characterizations inner two matters can be used to prove Newton's the second and third laws, to explain the structure of DNA, the shapes of Time Cone and topological sphere of moving trace. The force of two coupling matters has been expressed as a united formula $\stackrel{\rightharpoonup}{\boldsymbol{F}}=\boldsymbol{T}\left[\frac{d^{2} r}{d t^{2}}-r|\nabla \times \vec{u}|^{2}\right] \hat{\boldsymbol{r}}+\boldsymbol{T}\left(2|\nabla \times \vec{u}| \frac{d r}{d t}+r \frac{d|\nabla \times \vec{u}|}{d t}\right) \hat{\boldsymbol{\tau}}$, in which $r$ and $\vec{u}$ have the same origin of isolated system centre and can be instead of wave function to fit quantum mechanics, as shown in the literatures $[15,16]$. The interaction of coupling matters may be helpful to uniform the gravitation and electromagnetic force in future.

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Figure 1


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